Basic Counting

Finite Math

11 April 2019

Quiz

Give an example of a set.

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$$\begin{aligned} n(A) &= 5 \\ n(B) &= \end{aligned}$$

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$$\begin{aligned} n(A) &= 5 \\ n(B) &= 5 \\ n(A \cap B) &= 3 \\ n(A \cup B) &= 7 \\ n(A') &= 4 \\ n(A \cap B') &= 2 \\ n(\varnothing) &= 0 \end{aligned}$$

Example

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Let the universal set U be the set of positive integers less than or equal to 100. Let A be the set of multiples of 3 in U, and let B be the set of multiples of 5 in U.

(a) *Find $n(A \cap B)$, $n(A \cap B')$, $n(B \cap A')$, and $n(A' \cap B')$.*

(b) *Draw a Venn diagram with circles labeled A and B, indicating the numbers of elements in the subsets of part (a).*

Now You Try It!

Example

Let the universal set U be the set of positive integers less than or equal to 100. Let A be the set of multiples of 4 in U, and let B be the set of multiples of 7 in U.

(a) *Find $n(A \cap B)$, $n(A \cap B')$, $n(B \cap A')$, and $n(A' \cap B')$.*

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Solution

(a) $n(A \cap B) = 3$, $n(A \cap B') = 22$, $n(B \cap A') = 11$, and $n(A' \cap B') = 64$.

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Addition Principle

Suppose that there are 15 male and 20 female Physics majors at a university. How many total Physics majors are there?

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Theorem (Addition Principle for Counting)

For any two sets A and B,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Now You Try It!

Example

According to a survey of business firms in a certain city, 345 firms offer their employees group life insurance, 285 offer long-term disability insurance, and 115 offer group life insurance and long-term disability insurance. How many firms offer their employees group life insurance or long-term disability insurance?

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Example

According to a survey of business firms in a certain city, 345 firms offer their employees group life insurance, 285 offer long-term disability insurance, and 115 offer group life insurance and long-term disability insurance. How many firms offer their employees group life insurance or long-term disability insurance?

Solution

515

Multiplication Principle

Example

Suppose a store has 3 types of shirts, and in each type of shirt, they have 4 colors available. How many options are available?

Multiplication Principle

Theorem (Multiplication Principle for Counting)

1. *If two operations O_1 and O_2 are performed in order, with N_1 possible outcomes for the first operation and N_2 possible outcomes for the second operation, then there are*

$$N_1 \cdot N_2$$

possible combined outcomes of the first operation followed by the second operation.

2. *In general, if n operations $O_1, O_2, ..., O_n$ are performed in order, with possible number of number of outcomes $N_1, N_2, ..., N_n$, respectively, then there are*

$$N_1 \cdot N_2 \cdots N_n$$

possible combined outcomes of the operations performed in the given order.

Now You Try It!

Example

Suppose a 6-sided die and a 12-sided die are rolled. How many different possible outcomes are there?

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Suppose a 6-sided die and a 12-sided die are rolled. How many different possible outcomes are there?

Solution

72

More Multiplication Principle

Example

Suppose we have a list of 8 letters that we wish to make code words from. How many possible 4-letter code words can be made if:

(a) *letters can be repeated?*

(b) *no letter can be repeated?*

(c) *adjacent letters cannot be alike?*

Now You Try It!

Example

Suppose we have a list of 10 *letters that we wish to make code words from. How many possible* 5*-letter code words can be made if:*

(a) *letters can be repeated?*

(b) *no letter can be repeated?*

(c) *adjacent letters cannot be alike?*

Now You Try It!

Example

Suppose we have a list of 10 *letters that we wish to make code words from. How many possible* 5*-letter code words can be made if:*

(a) *letters can be repeated?*

(b) *no letter can be repeated?*

(c) *adjacent letters cannot be alike?*

Solution

(a) $100,000$*, (b)* $30,240$*, (c)* $65,610$